Implications from data with fuzzy attributes

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Abstract—We present an algorithm for generating a complete and non-redundant set of attribute implications from data table with fuzzy attributes. Rows and columns in the data table correspond to objects and attributes, respectively. An attribute implication is an expression which says that if an object satisfies a certain collection of attributes then it satisfies some other collection of attributes as well. The algorithm is based on reduction of the problem to the problem of generating fixed points of a fuzzy closure operator. We describe the theoretical insight, the algorithm, and examples.

I. INTRODUCTION AND PROBLEM SETTING

Tabular data describing objects and their attributes represents a basic form of data. Several methods have been designed to analyze object-attribute data. Among these, methods for obtaining if-then rules (implications) from data are of the most popular ones. For example, the well-known mining of association rules is an example of rule-extraction method, see e.g. [3], [8].

In our paper, we are interested in if-then rules generated from data with fuzzy attributes: rows and columns of data table correspond to objects and attributes, respectively. Table entries I(x,y) are truth degrees to which object x has attribute y. Unlike the classical case, we assume that I(x,y) is taken from a suitable scale L of truth degrees, e.g. L = [0,1]. We are interested in rules of the form “if A then B” (A ⇒ B), where A and B are collections of attributes, with the meaning: if an object has all the attributes of A then it has also all attributes of B. In crisp case, these rules were thoroughly investigated, see e.g. [10] for the first paper and [9] for further information and references. Our aim is basically to look at such if-then rules from the point of view of fuzzy logic. Our motivation is threefold: (1) in practice, attributes are usually fuzzy rather than bivalent; (2) non-logical attributes (like age, etc.) can be scaled to fuzzy attributes; (3) to investigate connections with related methods for processing of data with fuzzy attributes, particularly with formal concept analysis, e.g. [3], [8].

We study the following topics: appropriate tractable definition of if-then rules A ⇒ B and their semantics (validity degree etc.); directly related mathematical structures; the notion of semantic entailment of if-then rules with the aim to obtain a non-redundant basis of all valid rules; algorithms for generating bases.

II. PRELIMINARIES

As a set of truth degrees equipped with suitable operations (truth functions of logical connectives) we use a so-called complete residuated lattice with truth-stressing hedge. A complete residuated lattice with truth-stressing hedge is an algebra L = ⟨L, ∧, ∨, →, *, 0, 1⟩ such that ⟨L, ∧, ∨, 0, 1⟩ is a complete lattice with 0 and 1 being the least and greatest element of L, respectively; ⟨L, ⊗, 1⟩ is a commutative monoid (i.e. ⊗ is commutative, associative, and a ⊗ 1 = 1 ⊗ a for each a ∈ L); ⊗ and → satisfy so-called adjointness property:

\[ a ⊗ b ≤ c \iff a ≤ b → c \] (1)

for each a, b, c ∈ L; hedge * satisfies

\[ a^* ≤ a, \] (2)

\[ (a → b)^* ≤ a^* → b^*, \] (3)

\[ a^{**} = a^*, \] (4)

\[ ∨_{i∈I} a_i^* = (∨_{i∈I} a_i)^* \] (5)

for each a, b ∈ L, a_i ∈ L (i ∈ I). Elements a of L are called truth degrees. ⊗ and → are (truth functions of) “fuzzy conjunction” and “fuzzy implication”. Hedge * is a (truth function of) logical connective “very true”, see [11], [12]. Properties (2)–(5) have natural interpretations, e.g. (2) can be read: “if a is very true, then a is true”, (3) can be read: “if a → b is very true and if a is very true, then b is very true”, etc.

A common choice of L is a structure with L = [0,1] (unit interval), ∧ and ∨ being minimum and maximum, ⊗ being a left-continuous t-norm with the corresponding →. Three most important pairs of adjoint operations on the unit interval are:

Łukasiewicz:

\[ a ⊗ b = \max(a + b - 1, 0), \]

\[ a → b = \min(1 - a + b, 1), \]

Gödel:

\[ a ⊗ b = \min(a, b), \]

\[ a → b = \begin{cases} 1 & \text{if } a ≤ b, \\ b & \text{otherwise}, \end{cases} \]

Goguen (product):

\[ a ⊗ b = \begin{cases} 1 & \text{if } a ≤ b, \\ \frac{b}{a} & \text{otherwise}. \end{cases} \]

In applications, we usually need a finite linearly ordered L. For example, one can put L = \{a_0 = 0, a_1, \ldots, a_n = 1\} ⊆ [0,1] \ (a_0 < \cdots < a_n) with ⊗ given by a_k ⊗ a_l = a_{\max(k+l-n,0)} and the corresponding → given by a_k → a_l = a_{\min(n-k+l,n)}. Such an L is called a finite Łukasiewicz chain. Another possibility is a finite Gödel chain which consists of L and restrictions of Gödel operations on [0,1] to L. A special case of both of
these chains is the Boolean algebra with \( L = \{0, 1\} \) (structure of truth degrees of classical logic).

Two boundary cases of (truth-stressing) hedges are (i) identity, i.e. \( a^* = a \ (a \in L) \); (ii) globalization [14]:
\[
a^* = \begin{cases} 
1 & \text{if } a = 1, \\
0 & \text{otherwise.} 
\end{cases}
\]

Having \( L \) as our structure of truth degrees, we define usual notions: an \( L \)-set (fuzzy set) \( A \) in universe \( U \) is a mapping \( A: U \rightarrow L \), \( A(u) \) being interpreted as “the degree to which \( u \) belongs to \( A \)”. If \( U = \{u_1, \ldots, u_n\} \) then \( A \) can be denoted by \( A = \{a_1/u_1, \ldots, a_n/u_n\} \) meaning that \( A(u_i) \) equals \( a_i \) for each \( i = 1, \ldots, n \). Let \( L^U \) denote the collection of all \( L \)-sets in \( U \). The operations with \( L \)-sets are defined componentwise. For instance, intersection of \( L \)-sets \( A, B \in L^U \) is an \( L \)-set \( A \cap B \) in \( U \) such that \( (A \cap B)(u) = A(u) \wedge B(u) \) for each \( u \in U \), etc. Binary \( L \)-relations (binary fuzzy relations) between \( X \) and \( Y \) can be thought of as \( L \)-sets in the universe \( X \times Y \).

Given \( A, B \in L^U \), we define a subhood degree
\[
S(A, B) = \bigwedge_{u \in U} (A(u) \rightarrow B(u)),
\]
(10)
which generalizes the classical subhood relation \( \subseteq \) (note that unlike \( \subseteq, S \) is a binary \( L \)-relation on \( L^U \)). Described verbally, \( S(A, B) \) represents a degree to which \( A \) is a subset of \( B \). In particular, we write \( A \subseteq B \) if \( S(A, B) = 1 \). As a consequence, we have \( A \subseteq B \) if \( A(u) \leq B(u) \) for each \( u \in U \). In the following we use well-known properties of residuated lattices and fuzzy structures which can be found in monographs [3], [11].

III. Fuzzy attribute implications

A. Definition, validity, and basic properties

Fuzzy attribute implication (over attributes \( Y \)) is an expression \( A \Rightarrow B \), where \( A, B \in L^Y \) (\( A \) and \( B \) are fuzzy sets of attributes). The intended meaning of \( A \Rightarrow B \) is: “if it is (very) true that an object has all attributes from \( A \), then it also has all attributes from \( B \)”. The notions “being very true”, “to have an attribute”, and logical connective “if-then” are determined by the chosen \( L \).

For an \( L \)-set \( M \in L^Y \) of attributes, we define a degree \( ||A \Rightarrow B||_M \in L \) to which \( A \Rightarrow B \) is valid in \( M \):
\[
||A \Rightarrow B||_M = S(A, M)^* \rightarrow S(B, M).
\]
(11)

If \( M \) is the fuzzy set of all attributes of an object \( x \), then \( ||A \Rightarrow B||_M \) is the truth degree to which \( A \Rightarrow B \) holds for \( x \).

Fuzzy attribute implications can be used to describe dependencies in data tables with fuzzy attributes. Let \( X \) and \( Y \) be sets of objects and attributes, respectively, \( I \) be an \( L \)-relation between \( X \) and \( Y \), i.e. \( I \) is a mapping \( I : X \times Y \rightarrow L \). \( \langle X, Y, I \rangle \) is called a data table with fuzzy attributes. \( \langle X, Y, I \rangle \) represents a table which assigns to each \( x \in X \) and each \( y \in Y \) a truth degree \( I(x, y) \in L \) to which object \( x \) has attribute \( y \). If both \( X \) and \( Y \) are finite, \( \langle X, Y, I \rangle \) can be visualized as in Fig. 1

![Fig. 1. Data table with fuzzy attributes](image)

\[
A^\dagger \in L^Y \text{ (fuzzy set of attributes), } B^\dagger \in L^X \text{ (fuzzy set of objects) by}
\]
\[
A^\dagger(y) = \bigwedge_{x \in X} (A(x)^* \rightarrow I(x, y)),
\]
(12)
\[
B^\dagger(x) = \bigwedge_{y \in Y} (B(y) \rightarrow I(x, y)).
\]
(13)

We put
\[
B(X^*, Y, I) = \{ \langle A, B \rangle \in L^X \times L^Y | A^\dagger = B, B^\dagger = A \}
\]
and define for \( \langle A_1, B_1 \rangle, \langle A_2, B_2 \rangle \in B(X^*, Y, I) \) a binary relation \( \leq \) by \( \langle A_1, B_1 \rangle \leq \langle A_2, B_2 \rangle \) iff \( A_1 \subseteq A_2 \) (or, iff \( B_2 \subseteq B_1 \); both ways are equivalent). Operators \( \wedge, \vee \) form so-called Galois connection with hedge, see [6]. The structure \( B(X^*, Y, I) \) is called a fuzzy concept lattice induced by \( \langle X, Y, I \rangle \). The elements \( \langle A, B \rangle \) of \( B(X^*, Y, I) \) are naturally interpreted as concepts (clusters) hidden in the input data represented by \( I \). Namely, \( A^\dagger = B \) and \( B^\dagger = A \) say that \( B \) is the collection of all attributes shared by all objects from \( A \), and \( A \) is the collection of all objects sharing all attributes from \( B \). Note that these conditions represent exactly the definition of a concept as developed in the so-called Port-Royal logic; \( A \) and \( B \) are called the extent and the intent of the concept \( \langle A, B \rangle \), respectively, and represent the collection of all objects and all attributes covered by the particular concept. Furthermore, \( \leq \) models the natural subconcept-supercconcept hierarchy—concept \( \langle A_1, B_1 \rangle \) is a subconcept of \( \langle A_2, B_2 \rangle \) iff each object from \( A_1 \) belongs to \( A_2 \) (dually for attributes).

Now we define a validity degree of fuzzy attribute implications in data tables and intents of fuzzy concept lattices. First, for a set \( M \subseteq L^Y \) (i.e. \( M \) is an ordinary set of \( L \)-sets) we define a degree \( ||A \Rightarrow B||_M \in L \) to which \( A \Rightarrow B \) holds in \( M \) by
\[
||A \Rightarrow B||_M = \bigwedge_{M \in M} ||A \Rightarrow B||_M.
\]
(14)

Having \( \langle X, Y, I \rangle \), let \( I_x \in L^Y \) (\( x \in X \)) be \( L \)-set of attributes such that \( I_x(y) = I(x, y) \) for each \( y \in Y \). Described verbally, \( I_x \) is the \( L \)-set of all attributes of \( x \in X \), i.e. in \( \langle X, Y, I \rangle \), \( I_x \) corresponds to a row labeled \( x \). Clearly, we have \( I_x = \{1/x\} \) for each \( x \in X \).

A degree \( ||A \Rightarrow B||_{\langle X, Y, I \rangle} \in L \) to which \( A \Rightarrow B \) holds in (each row of) \( \langle X, Y, I \rangle \) is defined by
\[
||A \Rightarrow B||_{\langle X, Y, I \rangle} = ||A \Rightarrow B||_M,
\]
(15)
where \( M = \{I_x | x \in X \} \).

Denote
\[
\text{Int}(X^*, Y, I) = \{B \in L^Y | \langle A, B \rangle \in B(X^*, Y, I) \text{ for some } A\}
\]
the set of all intents of concepts of \( B(X^*, Y, I) \). Since \( M \in L^Y \) is an intent of some concept of \( B(X^*, Y, I) \) iff \( M = M^{11} \), we have \( \text{Int}(X^*, Y, I) = \{ M \in L^Y | M = M^{11} \} \).

A degree \(|A \Rightarrow B|_{B(X^*, Y, I)} \in L_0 \) to which \( A \Rightarrow B \) holds in (in terms of) \( B(X^*, Y, I) \) is defined by
\[
|A \Rightarrow B|_{B(X^*, Y, I)} = |A \Rightarrow B|_{\text{Int}(X^*, Y, I)}.
\] (16)

The following lemma connecting the degree to which \( A \Rightarrow B \) holds in \( (X, Y, I) \), the degree to which \( A \Rightarrow B \) holds in intents of \( B(X^*, Y, I) \), and the degree of subsethood of \( B \) in \( A^{11} \) was proved in [7].

**Lemma.** Let \( (X, Y, I) \) be a data table with fuzzy attributes. Then
\[
|A \Rightarrow B|_{(X, Y, I)} = |A \Rightarrow B|_{B(X^*, Y, I)} = S(B, A^{11})
\] (17)
for each fuzzy attribute implication \( A \Rightarrow B \).

**B. Implication bases**

The main obstacle to extract fuzzy concepts from data tables with fuzzy attributes is that large data tables and fine scales of truth degrees usually lead to large amounts of concepts which are then not graspable by our mind (it is unlikely to benefit from thousands of concepts, because we would have serious problems just trying to read them). It is then interesting to describe concepts as models of (possibly smaller) sets of fuzzy attribute implications.

Let \( (X, Y, I) \) be a data table with fuzzy attributes and let \( T \) be a set of fuzzy attribute implications. \( M \in L^Y \) is called a model of \( T \) if \( |A \Rightarrow B|_M = 1 \) for each \( A \Rightarrow B \in T \). The set of all models of \( T \) is denoted by \( \text{Mod}(T) \), i.e.
\[
\text{Mod}(T) = \{ M \in L^Y | M \text{ is a model of } T \}. \] (18)

A degree \(|A \Rightarrow B|_T \in L_0 \) to which \( A \Rightarrow B \) semantically follows from \( T \) is defined by
\[
|A \Rightarrow B|_T = |A \Rightarrow B|_{\text{Mod}(T)}. \] (19)

\( T \) is called complete (in \( (X, Y, I) \)) if \(|A \Rightarrow B|_T = |A \Rightarrow B|_{(X, Y, I)} \) for each \( A \Rightarrow B \). If \( T \) is complete and no proper subset of \( T \) is complete, then \( T \) is called a non-redundant basis. Note that the notion of completeness of \( T \) depends on a given data table with fuzzy attributes.

The following assertion shows that the models of a complete set of fuzzy attribute implications are exactly the intents of the corresponding fuzzy concept lattice.

**Theorem.** \( T \) is complete iff \( \text{Mod}(T) = \text{Int}(X^*, Y, I) \).

**Proof:** Let \( T \) be complete. Suppose \( M \in \text{Mod}(T) \). We have \(|M \Rightarrow M^{11}|_{\text{Int}(X^*, Y, I)} = S(M^{11}, M^{11}) = 1 \) by (17), i.e. \(|M \Rightarrow M^{11}|_T = 1 \) by completeness and (17). Since \( M \) is a model of \( T \), we have \(|M \Rightarrow M^{11}|_T = 1 \) which immediately gives \( 1 = S(M, M)^* \leq S(M^{11}, M) \), i.e. \( M^{11} \subseteq M \). That is, \( M \in \text{Int}(X^*, Y, I) \) proves that \( \text{Mod}(T) \subseteq \text{Int}(X^*, Y, I) \).

Conversely, if \( \text{Mod}(T) = \text{Int}(X^*, Y, I) \) then \(|A \Rightarrow B|_T = |A \Rightarrow B|_{\text{Int}(X^*, Y, I)} = |A \Rightarrow B|_{(X, Y, I)} \) by (17). Therefore, we are interested in finding non-redundant bases. First, a non-redundant basis \( T \) is a minimal set of implications which conveys, via the notion of semantic entailment, information about validity of attribute implications in \( (X, Y, I) \). In particular, attribute implications which are true (in degree 1) in \( (X, Y, I) \) are exactly those which follow (in degree 1) from \( T \). Second, non-redundant bases are promising candidates for being the minimal complete sets of attribute implications which describe the concept intents (and consequently, the whole fuzzy concept lattice).

**IV. Algorithm for getting non-redundant bases**

**A. Systems of pseudo-intents**

Given \( (X, Y, I) \), \( P \subseteq L^Y \) (a system of fuzzy sets of attributes) is called a system of pseudo-intents of \( (X, Y, I) \) if for each \( P \in L^Y \) we have:
\[
P \in P \text{ iff } P \neq P^{11} \text{ and } |Q \Rightarrow Q^{11}|_P = 1 \text{ for each } Q \in P \text{ with } Q \neq P.
\]

If * is globalization and if \( Y \) is finite, then for each \( (X, Y, I) \) there exists a unique system of pseudo-intents (this is not so for the other hedges in general). From now on, let \( Y \) be finite and let \( L \) be finite and linearly ordered. Moreover, for \( Z \in L^Y \) we put
\[
Z^+ = \bigcup \{ B \otimes S(A, Z)* \ | A \Rightarrow B \in T \text{ and } A \neq Z \},
\]
\[
Z^+ = Z,
\]
\[
Z^+ = (Z^{+})^{+}, \text{ for } n \geq 1,
\]
and define an operator \( c_{L^*} \) on \( L \)-sets in \( Y \) by
\[
c_{L^*}(Z) = \bigcup_{n=0}^{\infty} Z^+. \] (20)

**Theorem.** Let \( P \) be a system of pseudo-intents and put
\[
T = \{ P \Rightarrow P^{11} | P \in P \}. \] (21)

(i) \( T \) is non-redundant basis. Moreover, if * is globalization then \( T \) is minimal. (ii) If * is globalization then \( c_{L^*} \) is an \( L^* \)-closure operator, and
\[
\{ c_{L^*}(Z) | Z \in L^Y \} = P \cup \text{Int}(X^*, Y, I). \] (22)

**Proof:** For (i) see [7].

(ii) We check that \( c_{L^*} \) satisfies the following properties of \( L^* \)-closure operators
\[
Z \subseteq c_{L^*}(Z), \] (23)
\[
S(Z_1, Z_2)* \leq S(c_{L^*}(Z_1), c_{L^*}(Z_2)), \] (24)
\[
c_{L^*}(Z) = c_{L^*}(c_{L^*}(Z)), \] (25)
for each \( Z, Z_1, Z_2 \in L^Y \). (23) is obvious.

(24) Since * is globalization and \( T \) is defined by (21), we have \( Z^{+} = Z \cup \{ Q^{11} | Q \in P, Q \subseteq Z \} \). If \( S(Z_1, Z_2)* = 1 \) then \( Z_1 \subseteq Z_2 \), i.e. if \( Q \subseteq Z_1 \), then \( Q \subseteq Z_2 \) yielding \( Z_1^{+} \subseteq Z_2^{+} \), i.e. \( S(c_{L^*}(Z_1), c_{L^*}(Z_2)) = 1 \).
In order to show (25), it is sufficient to check \((cl_{T^*}(Z))^T \subseteq cl_{T^*}(Z)\). If \(Q \subseteq cl_{T^*}(Z)\) then \(Q \subseteq Z^{T^*}\) for some \(n \in \mathbb{N}_0\) (recall that \(L\) is finite and linearly ordered, and \(Y\) is finite). Hence, \(Q^{1\uparrow} \subseteq Z^{T^*}\). That is, \((cl_{T^*}(Z))^T = cl_{T^*}(Z)\) yielding \(cl_{T^*}(cl_{T^*}(Z))\). Altogether, \(cl_{T^*}\) is an \(L^*\)-closure operator.

\[ P \cup \text{Int}(X^*, Y, I) \subseteq \{cl_{T^*}(Z) | Z \in L^Y\} \] follows directly from properties of globalization. For the converse inclusion it suffices to show that if \(cl_{T^*}(Z) \neq cl_{T^*}(Z)^{1\uparrow}\) then \(cl_{T^*}(Z)\) is in \(P\). Consider \(Q \in P\) with \(Q \subseteq cl_{T^*}(Z)\). Then \(Q^{1\uparrow} \subseteq \{cl_{T^*}(Z)^{1\uparrow} = cl_{T^*}(Z)\}\), i.e. \(|Q| \neq |Q|\).

For general hedge *- operator \(cl_{T^*}\) need not satisfy the monotony condition (24). Consider a finite Łukasiewicz chain \(L\) with \(L = \{0, 0.1, 0.2, \ldots, 0.9, 1\}\) and let * be the identity \((a^* = a)\). Take \(T = \{(0.1/y) \Rightarrow \{1/y\}\}\). Obviously, \(cl_{T^*}(\{1/y\})(y) \geq \{(0.1/y)^T\}(y) = 0 \vee (1 \otimes (0.1 \rightarrow 0)) = 0.9\). On the other hand, \(cl_{T^*}(\{0.1/y\})(y) = 0.1\). Thus, we have \(\{1/y\} \subseteq \{0.1/y\}\), and \(cl_{T^*}(\{0.1/y\}) \not\subseteq cl_{T^*}(\{0.1/y\}\), i.e. \(cl_{T^*}\) is not monotone.

B. Algorithm for getting all (pseudo) intents

The previous theorem showed that for * being the globalization, we can get all intents and all pseudo-intents (of a given data table with fuzzy attributes) by computing the fixed points of \(cl_{T^*}\). This can be done with polynomial time delay using the fuzzy extension of Ganter’s algorithm for computing all fixed points of a closure operator, see [5].

Hence, if \(L\) is finite and linearly ordered residuated lattice with * being the globalization, one can compute all (pseudo) intents of \(\{X, Y, I\}\) \((Y\) being finite) using the following algorithm:

Input: \(\{X, Y, I\}\).
Output: \(\text{Int}(X^*, Y, I)\) (intents), \(P\) (pseudo-intents).

1. \(B := \emptyset\)
2. if \(B = B^{1\uparrow}\):
   - add \(B\) to \(\text{Int}(X^*, Y, I)\)
3. else:
   - add \(B\) to \(P\)
4. while \(B \neq Y\):
   - \(T := \{P \Rightarrow P^{1\uparrow} | P \in P\}\)
   - \(B := B^+\)
   - if \(B = B^{1\uparrow}\):
     - add \(B\) to \(\text{Int}(X^*, Y, I)\)
   - else:
     - add \(B\) to \(P\)

Note that \(B^+\) denotes the lexicographically smallest fixed point of \(L\)-closure operator \(cl_{T^*}\) which is a successor of \(B\). For more details on extended Ganter’s algorithm, we refer to [5].

V. ILLUSTRATIVE EXAMPLE AND REMARKS

Let \(L\) be a three-element Łukasiewicz chain such that \(L\) consists of \(L = \{0, 0.5, 1\} (0 < 0.5 < 1)\) endowed with \(\otimes, \rightarrow\) defined by (6), and let * be the globalization. The data table is given by Table I. The set \(X\) of objects consists of objects “Mercury”, “Venus”, . . . , \(Y\) contains four attributes: size of the planet (small/large), distance from the sun (far/near). The corresponding fuzzy concepts (clusters) extracted from this data table are identified in Table II, where each row represents a single concept. The subconcept-supercell concept hierarchy (fuzzy concept lattice) is depicted in Fig.2.

Concepts in Table II have natural interpretation. For instance concept 2 can be understood as anchor of “small planets far from sun”, concept 14 can be interpreted as anchor of “planets with average distance from sun”. Concepts 1 and 17 represent borderine concepts.

Before we present the non-redundant basis, let us introduce the following convention for writing finite \(L\)-fuzzy sets: we write \(\{\ldots, u, \ldots\}\) instead of \(\{\ldots, 1/u, \ldots\}\), and we also omit elements of \(U\) whose membership degree is zero. For example, we write \(\{u, 0.5/v\}\) instead of \(\{1/u, 0.5/v, 0/w\}\), etc.

The system \(P\) of pseudo-intents is the following

\[ P = \{\{s, 0.5/l, f\}, \{0.5/s, 0.5/n\}, \{l, f\}, \{0.5/l\}, \{f, 0.5/n\}, \{n\}\}. \]

Hence, the minimal non-redundant basis \(T\) defined by (21)
Experimental results have shown that the number of implications is usually (considerably) smaller than the number of concepts. However, the number of implications varies depending on density of the input data table (sparse tables can lead to relatively small amounts of concepts but large amounts of implications). Tables III and IV contain a summary of average number of (pseudo) intents of randomly generated data tables with 3, 5, or 7 attributes and with 5 up to 40 objects (columns labeled “B” contain average number of concepts, columns labeled “P” contain average size of the minimal bases, columns labeled “ratio” contain quotient of the previous values).

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![Fig. 2. Fuzzy concept lattice](image)